ECE231 Course Notes

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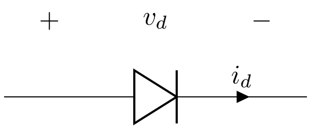
1 Diode's



1.1 Diode Characteristics

1.1.1 Forward Bias

$$i_d = I_s(e^{\frac{V}{nV_t}} - 1)$$

1.1.2 Reverse Bias

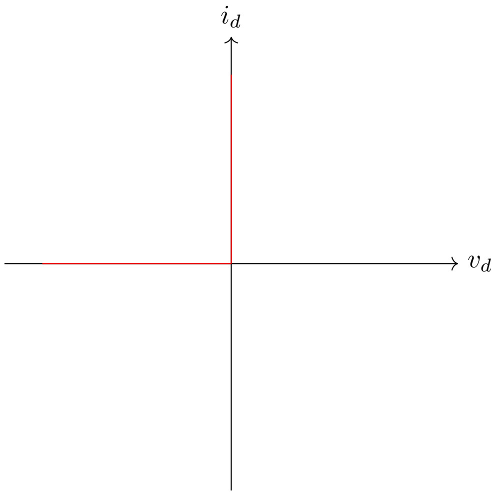
$$i_d = I_s(e^{\frac{V}{nV_t}} - 1)$$

$$i_d \approx I_s(0 - 1) = -I_s$$

1.1.3 Breakdown

$$v_d < v_{zk}$$

1.2 Ideal Diode IV Characteristic



1.3 Thermal Voltage

$$V_t = \frac{kT}{q}$$

where

- k = Boltzmann's Constant = $8.62 * 10^{-5}$eV/k = $1.38 * 10^{-23}$J/K
- T = Temperature in Kelvins
- q = Elementary Charge = $1.602 * 10^{-19}$C

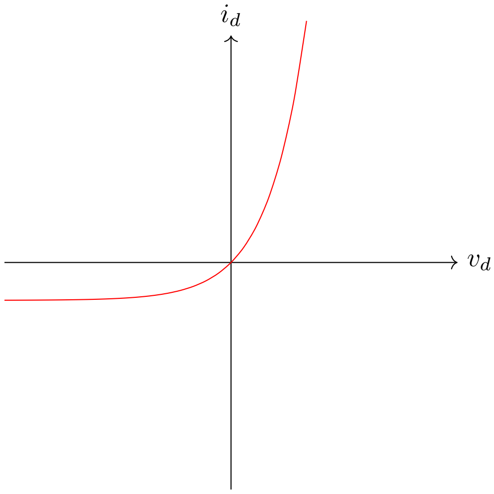
1.3.1 Approximation for V_t at room temperature

$$V_t \approx 25\text{mV}$$

1.4 Saturation Current

1.5 Diode Models

1.5.1 Exponential Model



$$i_d = I_s(e^{\frac{V}{nV_t}} - 1)$$

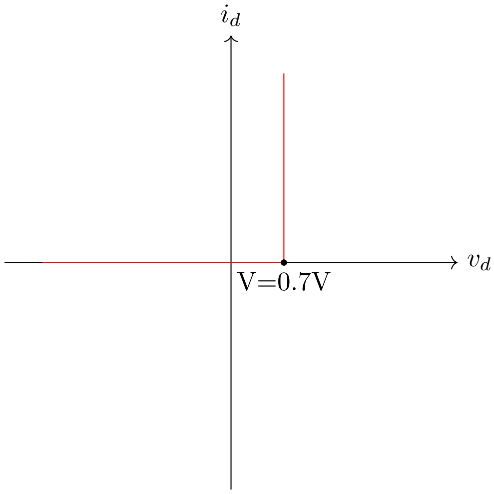
Where

- V_t: Thermal Voltage ($\approx 25mV$)
- k: Boltzmann's Constant ($= 8.62 \times 10^{-5} \quad \text{eV/K} = 1.38 \times 10^{-23} \quad \text{J/K}$)
- T: Temperature
- q: Elementary Charge
- I_s: Saturation Constant
- n: Ideality Constant

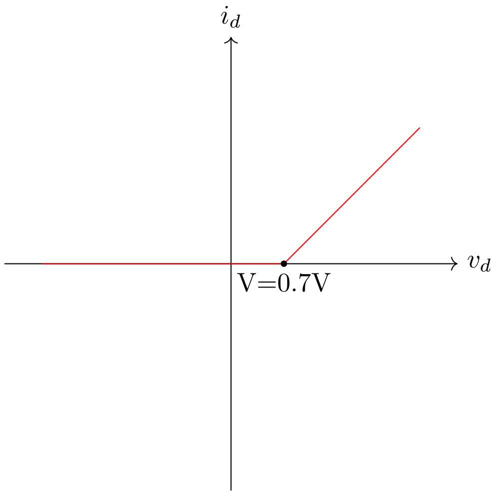
$$\frac{I_2}{I_1} = e^{\frac{v_2 - v_1}{nV_t}} = e^{\frac{\Delta v}{nV_t}}$$

$$\Delta v = nV_t \ln(\frac{I_2}{I_1})$$

1.5.2 Constant Voltage Drop Model (CVD)



1.5.3 Piecewise Linear Model (PWL)



1.6 Zener Diode (Reverse Bias Model)

At the Zener diode operating point Q (given by I_{ZT}, V_{ZT}),

$$V_Z = V_{Z0} + r_z I_Z$$

1.7 Photodiodes

$$i_p = R \cdot P$$

Where

- i_p = Photocurrent
- R = Responsivity
- P = Incident Light Power

2 Small Signal Analysis

$$i_D(t) = I_D(1 + \frac{v_d}{nv_t}) = I_D + \frac{1}{r_d}v_d$$

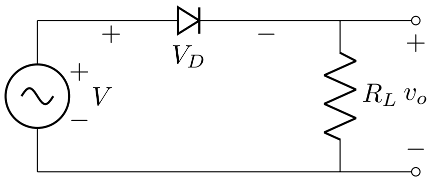
$$r_d = \frac{nv_t}{I_D}$$

3 Rectifiers

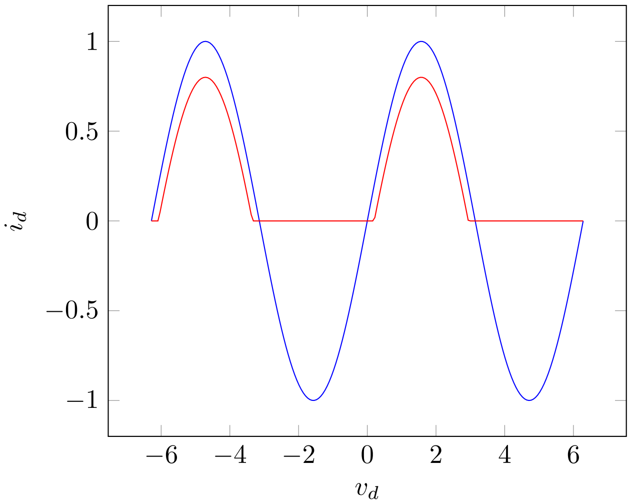
3.1 Peak Inverse Voltage (PIV)

Largest inverse voltage (reverse voltage) expected across diode. Good idea to select a diode that has a reverse breakdown voltage ($V_z k$) at least **50%** greater than expected PIV

3.2 Half Wave Rectifier



3.2.1 Half Wave Rectifier Waveform

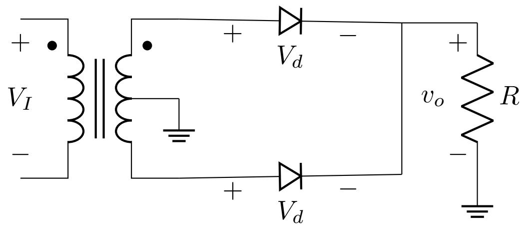


$$v_o = \begin{cases} 0, & v_s < V_D \\ v_s - V_D, & v_s \geq V_d \end{cases}$$

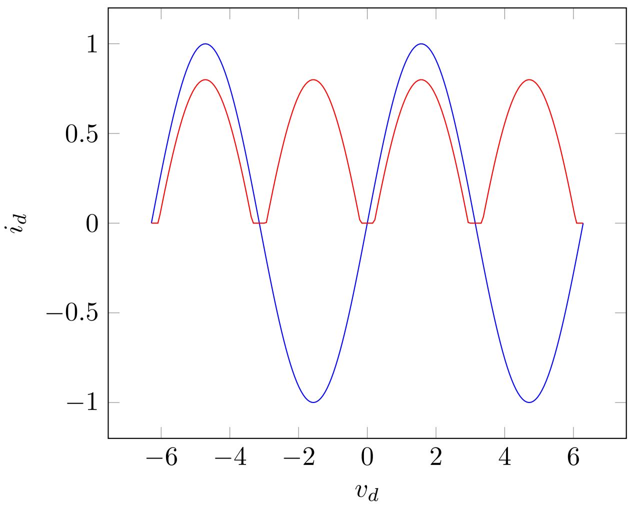
3.2.2 PIV

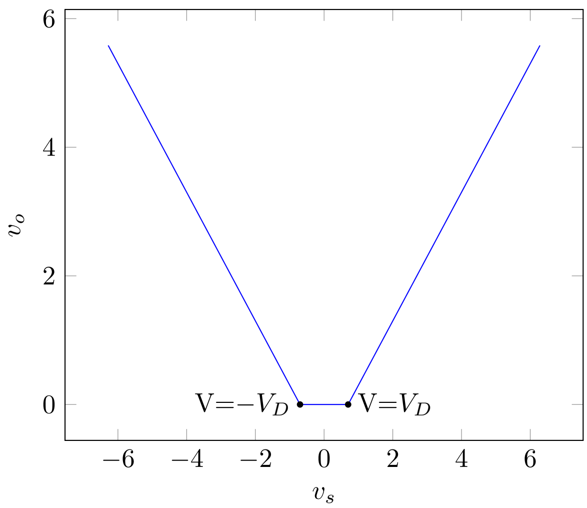
$$PIV = V_s$$

3.3 Full Wave Rectifier



3.3.1 Full Wave Rectifier Waveform

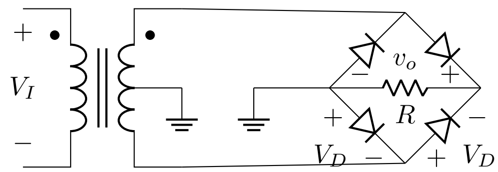




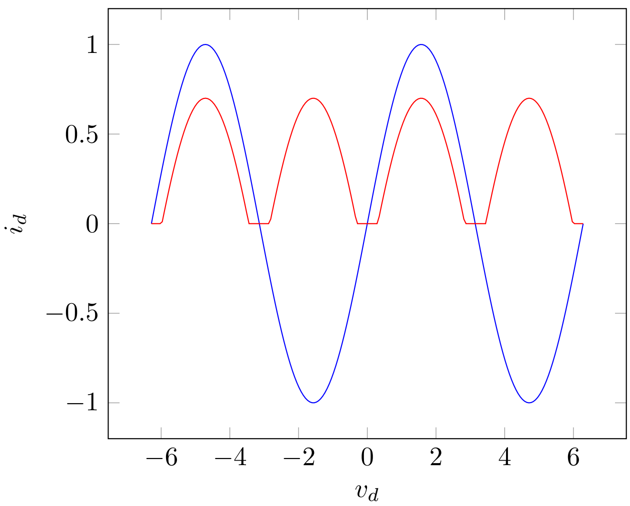
3.3.2 PIV

$$PIV = 2V_s - V_D$$

3.4 Bridge Rectifier



3.4.1 Full Bridge Rectifier Waveform

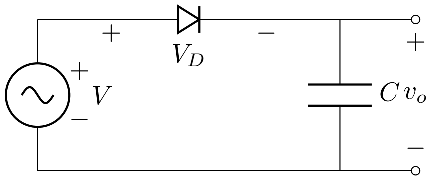


$$v_o = v_s - 2V_D$$

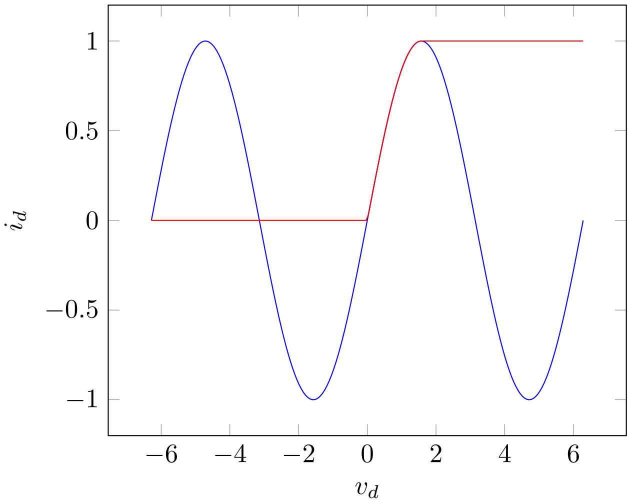
3.4.2 PIV

$$PIV = V_s - 2V_D + V_D = V_s - V_D$$

3.5 Peak Rectifier



3.5.1 Peak Rectifier Waveform



3.5.2 Ripple

For a half wave peak rectifier:

$$V_r = \frac{V_p}{fCR}$$

For a full wave peak rectifier:

$$V_r = \frac{V_p}{2fCR}$$

3.5.3 Conduction Interval

$$V_p \cos(\omega \Delta t) = V_p - V_r$$

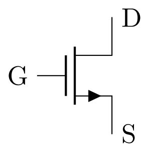
$$\text{Conduction Angle} = \omega \Delta t = \sqrt{\frac{2V_r}{V_p}}$$

$$\Delta t = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_p}}$$

4 MOSFET's

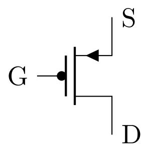
4.1 MOSFET Operating Regions

4.1.1 NMOS



Region	Source	Source Channel Inversion	Drain	Drain Channel Inversion
Cut-Off	$V_{GS} \leq V_{tn}$	No	$V_{GD} \leq V_{tn}$	No
Triode	$V_{GS} > V_{tn}$	Yes	$V_{GD} > V_{tn}; V_{DS} < V_{OV}$	Yes
Saturation	$V_{GS} > V_{tn}$	Yes	$V_{GD} \leq V_{tn}; V_{DS} \geq V_{OV}$	No

4.1.2 PMOS



Region	Source	Source Channel Inversion	Drain	Drain Channel Inversion
Cut-Off	$V_{SG} \leq \lvert V_{tp}\rvert$	No	$V_{DG} \leq \lvert V_{tp}\rvert$	No
Triode	$V_{SG} > \lvert V_{tp}\rvert$	Yes	$V_{DG} > \lvert V_{tp}\rvert; V_{SD} < \lvert V_{OV}\rvert$	Yes
Saturation	$V_{SG} > \lvert V_{tp}\rvert$	Yes	$V_{DG} \leq \lvert V_{tp}\rvert; V_{SD} \geq \lvert V_{OV}\rvert$	No

4.2 Voltage Measurements

4.2.1 Gate Source Voltage

$$V_{GS} = V_G - V_S$$

4.2.2 Drain Source Voltage

$$V_{DS} = V_D - V_S$$

4.2.3 Overdrive Voltage

$$V_{ov} = V_{GS} - V_t$$

4.2.4 Effective Voltage

$$V_{GS} - \frac{V_{DS}}{2} - V_t$$

4.3 Operating Region Equations

Region	NMOS	PMOS
Cut-off	$i_D = 0$	$i_D = 0$
Triode	$i_D = \mu_n C_{ox} \frac{W}{L}[(V_{GS} - V_{tn})V_{DS} - \frac{V_{DS}^2}{2}]$	$i_D = \mu_p C_{ox} \frac{W}{L}[(V_{SG} - \lvert V_{tp}\rvert)V_{SD} - \frac{V_{SD}^2}{2}]$
Saturation	$i_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{tn})^2[1 + \lambda V_{DS}]$	$i_D = \frac{1}{2}\mu_p C_{ox} \frac{W}{L}(V_{SG} - \lvert V_{tp}\rvert)^2[1 + \lvert\lambda\rvert V_{SD}]$

4.4 Oxide Capacitance

$$C_{ox} = \frac{\epsilon_0 \epsilon_r}{d} = \frac{\epsilon_{ox}}{d}$$

$$\epsilon_{r,silicon} = 3.9$$

$$t_{ox} = d = 4\text{nm} \quad \text{as an example}$$

$$\epsilon_{ox} = \epsilon_{r,silicon} \times \epsilon_0 = 3.9\epsilon_0 = 3.45 \times 10^{-11} F/m$$

4.5 Electron Drift Velocity

$$EDV = \mu_n |E| = \mu_n \frac{V_{DS}}{L}$$

4.6 Transconductance Parameter

$$k_n' = \mu_n C_{ox}$$

$$k_n = k_n'(\frac{W}{L}) = (\mu_n C_{ox})(\frac{W}{L})$$

4.7 Triode Region Resistance

$$r_{DS} = \frac{1}{g_{DS}} = \frac{1}{k_n V_{ov}} = \frac{1}{(\mu_n C_{ox})(\frac{W}{L})V_{ov}} = \frac{1}{(\mu_n C_{ox})(\frac{W}{L})(V_{GS} - V_t)}$$

5 BJT's

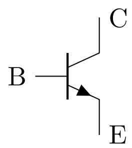
5.1 BJT Operating Regions

5.1.1 NPN

Region	V_{BE}	V_{BC}	EBJ	CBJ
Cut-Off	$V_{BE} \leq 0.7$		Reverse	Reverse
Active	$V_{BE} = 0.7$	$V_{BC} < 0.4$	Forward	Reverse
Saturation	$V_{BE} = 0.7$	$V_{BC} \geq 0.4$	Forward	Forward

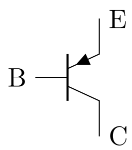
5.2 BJT Operating Models

5.2.1 NPN



Region	EBJ	CBJ	Assume	Verify
Active	Fwd	Rev	$V_{BE} = 0.7V$, $I_C = \alpha I_E = \beta I_B$	$I_B, I_E > 0$, $V_{BC} < 0.4V$, $V_{CE} > 0.3V$
Saturation	Fwd	Fwd	$V_{BE} = 0.7V$, $V_{BC} = 0.5V$	$I_B, I_E > 0$, $I_C/I_B = \beta_{forced} < \beta_{active}$, $V_{CE,sat} = 0.2V$
Cut-off	Rev	Rev	$I_E, I_B, I_C = 0$	$V_{BE} < 0.5V$, $V_{BC} < 0.4V$

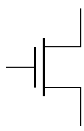
5.2.2 PNP



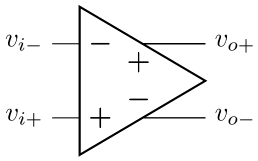
Region	EBJ	CBJ	Assume	Verify
Active	Fwd	Rev	$V_{EB} = 0.7V$, $I_C = \alpha I_E = \beta I_B$	$I_B, I_E > 0$, $V_{CB} < 0.4V$, $V_{EC} > 0.3V$
zwm Saturation	Fwd	Fwd	$V_{EB} = 0.7V$, $V_{CB} = 0.5V$	$I_B, I_E > 0$, $I_C/I_B = \beta_{forced} < \beta_{active}$, $V_{EC,sat} = 0.2V$
Cut-off	Rev	Rev	$I_E, I_B, I_C = 0$	$V_{EB} < 0.5V$, $V_{CB} < 0.4V$

6 Small Signal Transistor Models

6.1 MOSFET



7 Amplifiers



An amplifier that preserves the details of the signal waveform has the relationship

$$v_o(t) = Av_i(t)$$

Where

- v_o is the output voltage
- v_i is the input voltage
- A is the amplifier gain

7.1 Gain

7.1.1 Voltage Gain

$$A_v = \frac{v_o}{v_i}$$

$$A_{v,dB} = 20 \log |A_v| = 20 \log \left| \frac{v_o}{v_i} \right|$$

7.1.2 Current Gain

$$A_i = \frac{i_o}{i_i}$$

$$A_{i,dB} = 20 \log |A_i| = 20 \log \left| \frac{i_o}{i_i} \right|$$

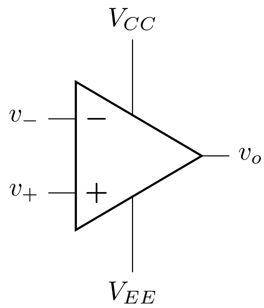
7.1.3 Power Gain

$$A_p = \frac{v_o}{v_i} \frac{i_o}{i_i}$$
$$A_p = A_v A_i$$

$$A_{p,dB} = 10 \log A_p$$

7.2 Amplifier Supply

7.2.1 BJT

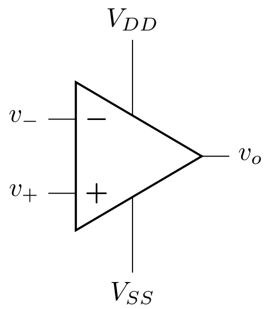


where

- V_{CC} is typically connected to collector
- V_{EE} is typically connected to emitter

$$V_{EE} < V_o < V_{CC}$$

7.2.2 MOSFET



where

- V_{DD} is typically connected to drain
- V_{SS} is typically connected to source

7.3 Amplifier Current

$$i_C(t) = I_C + i_c(t)$$

Where

- i_C is the sum of DC and AC currents
- I_C is the DC current (representing operating point)
- i_c is the AC current (representing small signal current)

7.4 Voltage Amplifier

7.4.1 Output Voltage

$$v_o = A_{vo} v_i \frac{R_L}{R_L + R_o}$$

7.4.2 Voltage Gain

$$A_v \equiv \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o}$$

7.4.3 Open Circuit Voltage Gain

When $R_L = \infty$,

$$A_v = A_{vo}$$

7.4.4 Overall Voltage Gain

$$\frac{v_o}{v_s} = A_{vo} \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o}$$

7.4.5 Transconductance Gain

$$A_{vo} = G_m R_o$$

$$G_m = \frac{i_o}{v_i}$$

$$A_{vo} = \frac{R_m}{R_i}$$

8 Transistor Amplifiers

8.1 Voltage Gain

$$A_v \equiv \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o}$$

8.2 OVerall Voltage Gain

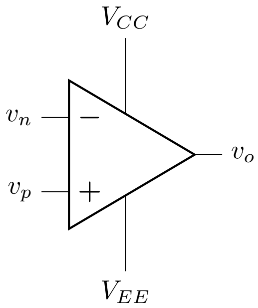
$$G_v \equiv \frac{v_o}{v_{sig}}$$

8.3 Common-Source Amplifier (MOSFET)

Most widely used amplifier configuration (of three basic transistor amp configurations)

8.4 Common-Emitter Amplifier (BJT)

9 Operational Amplifiers



9.1 Ideal Op Amp Characteristics

$$v_o = A(v_p - v_n)$$

Where A is the **differential gain** or **open-loop gain**

$$A = \infty$$

$$R_{in} = \infty$$

$$R_{out} = 0$$

9.1.1 Common-Mode Rejection

Under ideal conditions, if $v_n = v_p = 1$, then $v_o = 0$

9.1.2 Differential and Common-Mode Signals

Differential Input Signal V_{Id}:

$$V_{Id} = v_p - v_n$$

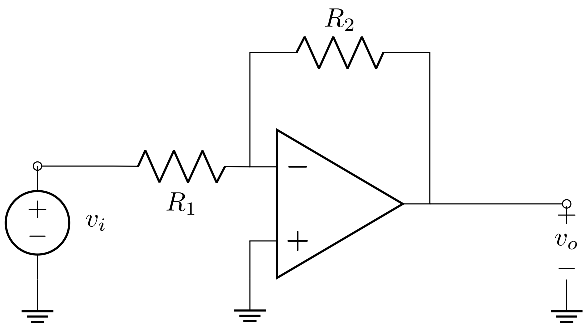
Common-mode Input Signal V_{Icm}:

$$V_{Icm} = \frac{1}{2}(v_p + v_n)$$

$$v_p = v_{Icm} + V_{Id}/2$$

$$v_n = v_{Icm} - V_{Id}/2$$

9.2 Inverting OpAmps



9.2.1 Inverting OpAmp Characteristics

$$i_n = i_p = 0$$

$$v_n = v_p$$

9.2.2 Inverting OpAmp Gain

$$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

9.3 NonIdeal OpAmp Characteristics

9.3.1 Input Offset Voltage

For an OpAmp in closed-loop inverting configuration

$$V_o = \left(1 + \frac{R2}{R1}\right) V_2 + \left(-\frac{R_2}{R_1}\right) V_1$$

For an offset voltage V_{os} applied at the V_p terminal

$$V_o = V_{os} \left(1 + \frac{R2}{R1}\right)$$

9.3.2 Input Bias/Offset Currents

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

$$I_{os} = |I_{B1} - I_{B2}|$$

For an OpAmp in closed-loop inverting configuration with Input Bias Currents

$$V_o = R_2 I_{B1} - \left(1 + \frac{R2}{R1}\right)(R_3 I_{B2})$$

9.3.3 Finite Open-Loop Gain

$$A(s) = \frac{A_0}{1 + s/\omega_b} \quad \longrightarrow \quad A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b}$$

9.3.4 Finite Bandwidth

$$A_o \omega_b = \omega_t$$

$$\left(1 + \frac{R_2}{R_1}\right) \omega_{3dB} = \omega_t$$

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}}$$

9.4 Large Scale Phenomenon

9.4.1 Slew Rate

$$SR = \frac{dv_o}{dt}|_{max}$$

$$\tau = \frac{1}{\omega_t}$$