1 Neuroanatomy

1.1 Neural Circuits

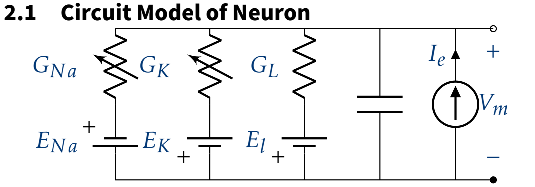
- **Reflex**: Bypasses brain (reflexes)
- **Divergent**: One to many
- **Convergent**: Many to one
- **Reverberating**: Positive feedback, always on (breathing circuitry)
- **Parallel After-Discharge**: convergent + divergent, visual integration, lag compensation

1.2 Brain Membranes/Barriers

Membranes	Barriers
Dura Mater	Blood-CSF
Arachnoid Layer	Blood-Brain (BBB)
Pia Mater	Arachnoid Barrier

2 Electrophysiology

2.1 Circuit Model of Neuron



where g is the conductance, E is the equilibrium potential, C is the capacitance, V_m is the resting potential

2.2 Nernst Planck Equation

$$\bar{J}_P = -D_p\left(\nabla C_p + \frac{Z_p C_p F}{RT}\nabla\phi\right)$$

Where $\bar{J}$ is the ionix flux , R is the gas constant, T is the absolute temperature, F is Faraday's constant

2.3 Equilibrium

At equilibrium, $\bar{J}_d + \bar{J}_e = \bar{J}_p = 0$. Nernst-planck:

$$\nabla C_p = -\frac{Z_p C_p F}{RT}\nabla\phi \qquad \frac{dC_p}{C_p} = \frac{Z_p C_p F}{RT}d\phi$$

Then the equilibrium potential across the membrane is

$$V_m^{eq} = -\frac{RT}{Z_p F}\ln\left(\frac{Cp_i}{Cp_e}\right)$$

where Cp_i, Cp_e are the intracellular and extracellular ion concentrations, respectively.

3 Signal Processing

3.1 Signals

Term	Definition
Linear	$A \cdot g(x) = g(Ax)$ $f(x) + f(y) = f(x+y)$
Stationary	constant statistical properties
Time-Invariant	statistical properties do not change with time

3.1.1 Convolution

$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

3.1.2 Expected Value

$$E[x] = \int_{-\infty}^{\infty} x \cdot PDF_X(x)dx$$

3.1.3 Cross-Correlation

$$R_{XY}[x,y] = E[X[x]Y[y]]$$

$$(f \star g)(t) := \int_{-\infty}^{\infty} f^*(\tau)g(t-\tau)d\tau$$

3.1.4 Auto-Correlation

$$R_{XX}[x,y] = E[X[x]X[y]]$$

3.2 Signal Processing Equations

Mean: $\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n$

$\bar{x}(t) = \frac{1}{T}\int_0^T x(t)dt$

RMS: $x_{rms} = \left[\frac{1}{N}\sum_{n=1}^{N} x_n^2\right]^{\frac{1}{2}}$

$x_{rms}(t) = \left[\frac{1}{T}\int_0^T x(t)^2 dt\right]^{\frac{1}{2}}$

Variance: $\sigma^2 = \frac{1}{N-1}\sum_{n=1}^{N}(x_n - \bar{x})^2$

$\sigma^2 = \frac{1}{T}\int_0^T (x(t) - \bar{x})^2 dt$

Std. Dev: $\sigma = \sqrt{\sigma^2}$

SNR: $SNR_{dB} = 20\log\frac{s_{rms}}{n_{rms}}$

$SNR_{linear} = 10^{\frac{SNR_{dB}}{20}}$

Power spectrum: $PS(f) = \frac{1}{T}\int_0^T r_{xx}(t)e^{-j2\pi n f_1 t}dt$

$PS[n] = \sum_{n=0}^{N-1} r_{xx}[k]e^{-j2\pi n f_1 T k}$

3.3 Fourier Transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

3.4 CTDT

$$H[z] = \frac{Y[z]}{X[z]} = \frac{b[0] + b[1]z^{-1} + \ldots + h[k]z^{-k}}{1 + a[1]z^{-1} + \ldots + a[l]z^{-l}}$$

3.5 Averaging

Averaging can reduce noise when multiple observations are possible

$$\bar{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N}\sigma_n^2 \qquad \overline{\sigma_{avg}} = \frac{\sigma}{\sqrt{N}}$$

3.6 Sampling

where $f_s = \frac{1}{T_s}$ is the sample frequency, T_s is the sample period, n is the sample index, $t = nT_s$

3.6.1 Shannon Sampling Theory

$$f_{Nyquist} = \frac{f_s}{2}$$

3.6.2 Aliasing

Aliasing occurs when $f_{max} > \frac{f_s}{2}$

3.7 Power Spectrum

Since autocorrelation has even symmetry,

$$PS(f) = \frac{1}{T}\int_0^T r_{xx}(t)\cos(2\pi m f t)dt \quad m = 0, 1, 2, \cdot$$

$$PS[n] = \sum_{n=0}^{N-1} r_{xx}[k]\cos\left(\frac{2\pi n m}{N}\right) \quad m = 0, 1, 2, \cdot, N$$

PS can be calculated from FT: $PS(f) = |X(f)|^2$

4 Filters

$$f_c = 3dBpoint = 0.707 \cdot G$$

where G is the filter gain

4.1 Quantization Error

$$q = \frac{V_{max}}{2^b - 1}\text{volts}$$

Where q is the quantization step, b is the # of bits. If the quant noise is evenly distributed $\in [\frac{-q}{2}, \frac{q}{2}]$

$$\sigma^2 = \int_{-q/2}^{q/2}\frac{e^2}{q}de = \left.\frac{e^3}{3q}\right|_{-q/2}^{q/2} = \frac{q^2}{12}$$

4.2 Dynamic Range

$$DR = 2^b - 1 \quad DR_{dB} = 20 \cdot \log_{10}(2^b - 1)$$

4.3 Finite Impulse Response (FIR)

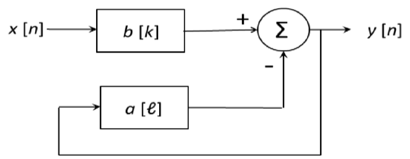
$$y[n] = \sum_{k=0}^{N-1} b[k]x[n-k] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k]$$

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1}\delta[n-k] = \begin{cases}\frac{1}{N} & 0 \le n < N \\ 0 & n < 0, n \ge N\end{cases}$$

4.4 Infinite Impulse Response (FIR)

$$y[n+1] = Ay[n] + Bx[n]$$

4.5 Fundamental Filter Equation



$$y[n] = \sum_{k=0}^{K-1} b[k]x[n-k] -]\sum_{l=1}^{L} a[l]y[n-l]$$

4.6 Other FIR Filters

By modifying $b[k]$, create other filters from FIR:

$$\text{High pass}: \ b[k] = \begin{cases}\frac{-\sin(2\pi k f_c)}{\pi k} & k \ne 0 \\ 1 - 2f_c & k = 0\end{cases}$$

$$\text{Band pass}: \ b[k] = \begin{cases}\frac{\sin(2\pi k f_h)}{\pi k} - \frac{\sin(2\pi k f_l)}{\pi k} & k \ne 0 \\ 2(f_h - f_l) & k = 0\end{cases}$$

$$\text{Band Stop}: \ b[k] = \begin{cases}\frac{\sin(2\pi k f_l)}{\pi k} - \frac{\sin(2\pi k f_h)}{\pi k} & k \ne 0 \\ 1 - 2(f_h - f_l) & k = 0\end{cases}$$

Note: Order of highpass/bandstop should be even, odd coeff's

Moving Average (FIR)	only $b[n]$
Autoregressive Moving Average (ARMA/IIR)	$a[n], b[n]$
Autoregressive (AR)	only $a[n]$, $b[k] = 1$

4.7 Wiener Filter

$$y[k] = \sum_{k=0}^{N-1} b[k]x[n-k] \qquad e[k] = d[k] - y[k]$$

where $d[k]$ = desired response, and $e[k]$ = error. $b[k]$ tuned to reduce $e^2[n]$ (least mean square), $y[k] \to d[k]$

4.8 Adaptive Filtering

$$y'[k] = \sum_{k=0}^{N-1} b_n[k]x[n-k] \qquad y[k] = d[k] - y'[k]$$

variable frequency char., $b_n[k]$ changes over time

5 Frequency Analysis

5.1 Short-Term Fourier Transform

$$X(t, f) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-j2\pi m f t}dt$$

$$X[m, k] = \sum_{n=1}^{N} x[n](W[n-k]e^{-j\omega m/n})$$

The STFT has a time-frequency uncertainty limit given by $BT \ge \frac{1}{4\pi}$, where B is bandwidth resolution and T is time resolution.

5.2 Continuous Wavelet Transform

$$W(a, b) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{|a|}}\Psi^* \cdot \left(\frac{t-b}{a}\right)dt$$

$b = 0, a = 1 \longrightarrow$ mother wavelet. Tradeoff between time/frequency: $\Delta\omega_\psi(a)\Delta t_\psi(a) = \Delta\omega_\psi\Delta t\psi = c \ge 0.5$ for constant c.

5.2.1 Morlet Wavelet

$$\Psi(t) = e^{-t^2}\cos\left(\pi\sqrt{\frac{2}{\ln 2}}t\right)$$

5.3 Discrete Wavelet Transform

Restricts CWT to power of 2, downsamples data.

6 Multivariate Analysis

6.1 Linear Transformation

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ M \\ y_M(t)\end{bmatrix} = W\begin{bmatrix} x_1(t) \\ x_2(t) \\ M \\ x_M(t)\end{bmatrix} \qquad y_i(t) = \sum_{j=1}^{M} w_{ij}x_j(t), i = 1, K$$

Interpretation: rotation of the data set.

6.2 Principle Component Analysis

A series of linear transformations until all variables are uncorrelated. Provides information on dimensionality of dataset, and fewest # of variables w/ most essential info. Scree plot shows λ (eigenvalues) of PCA. PCA finds directions of maximum variance.

6.3 Independent Component Analysis

$$x = As = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} = A \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_N(t) \end{bmatrix} \qquad s = A^{-1}x$$

Where A is the mixing matrix, s are hidden (source) signals, and x are measured signals. ICA can only be applied to non-Gaussian signals. Cannot determine variances, energies, and amplitudes of sources. ICA finds directions of maximum independence

7 Nonlinear Dynamics

Phase-space plot can give important information about a system from signals. Attractors are tendency of a system at equilibrium.

Exponential Divergence: A property of chaotic systems where trajectories from close initial conditions are wildly different.

7.1 Delay Embedding

Time series $x[n]$ (length N) can be reconstructed into multidimensional time series $y[n_d, k]$ of k dimensions from 1 to m. Each delayed vector n_d comes from $x[n]$ delayed by τ

$$y[n_d, k] = x[n + (k-1)\tau, k] \ldots x[N - (m-1)\tau, m]$$

m should be twice D (D = true dimension of system). τ can be any number, but in practice not too large or small.

8 Information Theory

8.1 Entropy

Entropy is defined as the number of bits needed to encode information uniquely

$$H_x = -\sum_m p(x)\log_2 p(x)$$

Max entropy: $p(x) = \frac{1}{N}\ \forall x \Rightarrow H_x = 1$

Min entropy: $p(x) = 1 \Rightarrow H_x = 0$

For non-stochastic signals, entropy is a measurement of signal regularity. Sine wave is regular $\Rightarrow$ low entropy.

8.1.1 Coin Example of Entropy

Biased coin (7500 heads, 2500 tails)

$$H = -(\log_2(0.75) * 0.75 + \log_2(0.25) * 0.25) = 0.8113$$

Unbiased coin (5000 heads, 5000 tails)

$$H = -(\log_2(0.5) * 0.5 + \log_2(0.5) * 0.5) = 1$$

8.2 Approximate Entropy (ApEn)

Can account for nonlinear properties unlike spectral/probabilitic methods. Bin segments of samples w/ length m, delay τ. Sample with similar, repeating segments $\Rightarrow$ high regularity, low entropy. **Sample Entropy (SampEn)** refinement of ApEn (more stable, remove bias) due to self match.

8.3 Mutual Information

$$MI_{xy} = H_x + H_y - H_{xy} = \sum_{mk} p(x,y)\log_2 \frac{p(x,y)}{p(x)p(y)}$$

$$H_{xy} = -\sum mk\, p(x,y)\log_2 p(x,y)$$

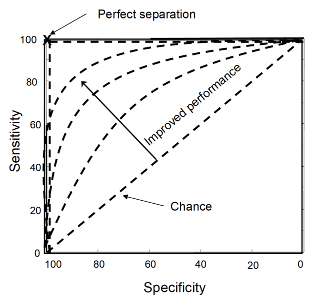
Need to estimate $p(x,y)$ with cells (2D histogram), each cell contains count of pairs of values

9 Machine Learning

9.1 Measures of Accuracy & Metrics

$$Sens. = \frac{TP}{TP+FN} \qquad Spec. = \frac{TN}{TN+FP}$$

ROC curve:



9.2 Kernel Machines

Transform data into higher-dimensional space, generate linear boundary. This is always possible for sufficient dimensions (Cover's Theorem)

9.3 Support Vector Machines

Finds points closest to boundary (support vectors), get hyperplane between support vectors. AKA Maximum Margin classifiers.

9.4 Cluster Analysis

k-nearest neighbors: Take average of k nearest training points, assign test point. k-means clustering: Reduce dataset to k prototype centers, classify test points based on closest prototype.

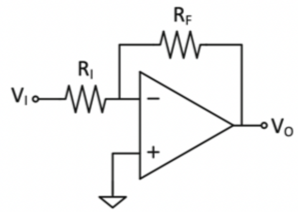
10 Electrical Neural Interfaces

10.1 Amplifiers

$$V^+ - V^- = \frac{V_o}{A}$$

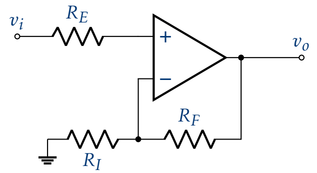
Ideal: $R_{in} = \infty$, $R_o = 0$, $A = \infty$

10.2 Inverting Amplifiers



$$i_i = i_f \Rightarrow \frac{V_i - 0}{R_i} = \frac{0 - V_o}{R_f} \Rightarrow \frac{V_o}{V_i} = -\frac{R_f}{R_i}$$

10.3 Neural Signal Amplification w/ Electrode



$$i_i = i_f \Rightarrow \frac{V^-}{R_i} = \frac{V_o - V_i}{R_f} \Rightarrow \frac{V_o}{V_i} = \frac{R_f + R_i}{R_i}$$

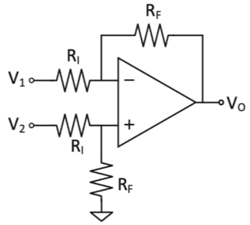
10.4 Common Mode Rejection

$$V_{CM} = \frac{V^+ + V}{2} \qquad V_{DM} = V^+ - V^-$$

$$V^+ = V_{CM} + \frac{V_{DM}}{2} \qquad V^- = V_{CM} - \frac{V_{DM}}{2}$$

Common Mode Rejection Ratio: $CMRR = \frac{A_{V,DM}}{A_{V,CM}}$

10.5 Differential Amplifier



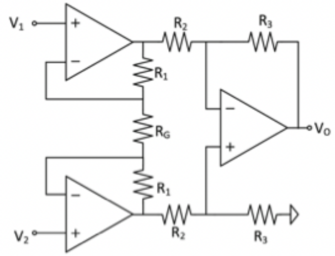
$$\frac{V_{CM} - V_-}{R_i} = \frac{V_- - V_o}{R_f} \qquad \frac{V_{CM} - V_+}{R_i} = \frac{V_+}{R_f} \qquad V_+ = V_-$$

$$\frac{V_{DM} - V_-}{R_i} = \frac{V_- - V_o}{R_f} \qquad \frac{V_{DM} - V_+}{R_i} = \frac{V_+}{R_f} \qquad V_+ = V_-$$

$$A_{V,DM} = -\frac{R_f}{R_i} = -\frac{R_2}{2R_1} - \frac{R_4(R_1 + R_2)}{2R_1(R_3 + R_4)}$$

Want $R_1 = R_3 = R_i$, $R_2 = R_4 = R_f$

10.6 Instrumentation Amplifier

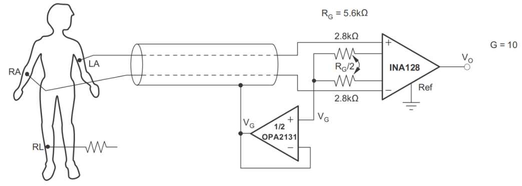


$$V_1 = V_2 = V_{CM} \qquad V_1' = V_2' = V_{CM} \Rightarrow I_G = 0, V_o = 0$$

$$I_G = \frac{V_1' - V_2'}{R_G} \qquad V_o1 - V_o2 = V_{DM}\left(1 + \frac{2R_1}{R_G}\right)$$

$$A_{V,DM} = \left(1 + \frac{2R_1}{R_G}\right)\left(-\frac{R_3}{R_2}\right)$$

10.7 Shielding

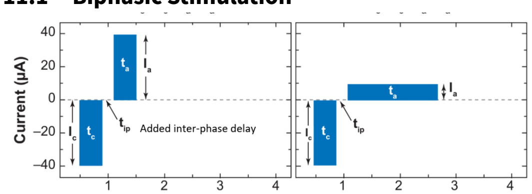


11 Stimulation

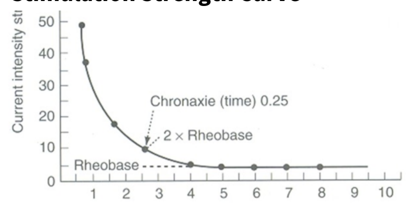
Voltage: Poor safety, good efficiency ($I_{Z_L} = V_{Z_L}/Z_L$)

Current: Good accuracy (safety), limited efficiency ($V_{Z_L} = I_{Z_L} \cdot Z_L$)

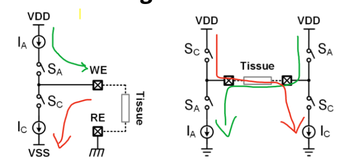
11.1 Biphasic Stimulation



11.2 Stimulation Strength Curve



11.3 Electrode Configuration



12 Constants and Units

Constant	Value
Gas Constant	$8.3145 \frac{J}{mol \cdot K}$
Faraday Constant	$96485.3321 \frac{C}{mol}$